Critical Behavior of Damping Rate for Plasmon with Finite Momentum in ϕ^4 Theory

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Applying thermal renormalization group (TRG) equations to ϕ^4 theory with spontaneous breaking symmetry, we investigate the critical behavior of the damping rate for the plasmons with finite momentum at the symmetry-restoring phase transition. From the TRG equation the IR cutoff provided by the external momentum leads to that the momentum-dependent coupling constant stops running in the critical region. As the result, the critical slowing down phenomenon reflecting the inherently IR effect doesn't take place at the critical point for the plasmon with finite external momentum.

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As well known, at high temperature the spontaneous-breaking symmetry in scalar field theory can be restored through phase transition. At finite temperature the effective degrees of the freedom of the scalar field are collective modes which is interpreted as quasiparticles, the so-called plasmons. The plasmon possess finite thermal mass which is generated dynamically by the interactions among the fundamental degrees of freedom of field. The thermal mass plays an important role at the phase transition for restoring spontaneous-breaking symmetry. The width of the spectral density of the plasmon is described by damping rate defined as [1]

$$\gamma_{\mathbf{k}}(T) \equiv \frac{\mathrm{Im}\Sigma(\omega_{\mathbf{k}}, \mathbf{k})}{2\omega_{\mathbf{k}}}\,, \tag{1}$$

where $\mathrm{Im}\Sigma$ is the imaginary part of the self energy, $\omega_{\mathbf{k}}^2 = |\mathbf{k}|^2 + m_p^2(T)$, $m_p(T)$ the plasmon mass and $\mathbf{k}$ the plasmon momentum. As shown by Weldon in Ref. [1], if the plasma is slightly out of thermal equilibrium, then $\gamma_{\mathbf{k}}(T)$ gives half the relaxation rate (or the inversion of the relaxation time) of the quasiparticle distribution function to its equilibrium value,

$$\frac{d\delta n_{\mathbf{k}}}{dt} = -2\gamma_{\mathbf{k}}(T)\delta n_{\mathbf{k}}\,, \tag{2}$$

where $\delta n_{\mathbf{k}}$ is the deviation of the distribution function from equilibrium, $\delta n_{\mathbf{k}} = n_{\mathbf{k}} - n_{\mathbf{k}}^{eq}$.

In Refs. [2] and [3] Parwani and Jeon investigated the damping rate of the plasmon at rest in massless ϕ^4 theory. In our previous paper [4], we generalized their work to discuss the damping rate of plasmon with finite momentum. All these works don't take into account the effect of thermal renormalization group on the coupling constant. As shown by Pietroni [5], for plasmon at rest the damping rate is divergent at critical temperature of the phase transition for restoring symmetry in ϕ^4 theory with spontaneous-breaking symmetry. This critical behavior contradicts the critical slowing down law. The singularity of the damping rate of the plasmon at critical point can be cured by taking into account the running coupling constant with temperature from thermal renormalization group (TRG) equation. In this paper, we will generalize Pietroni's work to investigate the critical behavior of the damping rate for the plasmons with finite momentum in general case.

We consider following Lagrangian in ϕ^4 theory with spontaneous-breaking symmetry,

$$\mathcal{L}_0 = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \frac{1}{2}\mu_0^2\phi^2 + \frac{\lambda_0}{4!}\phi^4\,, \tag{3}$$

where $\mu_0^2 < 0$ and λ_0 is coupling constant. A consistent determination of the plasmon damping rate requires the resummation of hard thermal loop [6]. The induced thermal mass correction resulting from the hard thermal loop (the tadpole diagram) reads

$$\Delta m_T^2 = \frac{\lambda_0 T^2}{24}\,. \tag{4}$$

The plasmon mass can be defined as

$$m_p^2 = \mu_0^2 + \frac{\lambda_0 T^2}{24}\,. \tag{5}$$

Notice that the thermal mass correction Δm_T^2 is positive. Although $\mu_0^2 < 0$, at enough high temperature $T > T_c$ the plasmon mass becomes positive, the spontaneous-breaking symmetry is restored. The critical temperature T_c can be expressed as

$$T_c = \sqrt{-\frac{24\mu_0^2}{\lambda_0}}\,. \tag{6}$$

In ϕ^4 theory, the resummation of the hard thermal loop is much easier since the hard thermal loop is just a momentum-independent real constant. The effects from hard thermal loop can be resummed by defining an effective Lagrangian through [4]

$$\begin{aligned}\mathcal{L} &= (\mathcal{L}_0 + \frac{1}{2}\Delta m_T^2\phi^2) - \frac{1}{2}\Delta m_T^2\phi^2 \\ &= \mathcal{L}_{eff} - \frac{1}{2}\Delta m_T^2\phi^2\,,\end{aligned} \tag{7}$$

and treating the last term as an additional interaction. This effective Lagrangian defines an effective propagator

$$\Delta(k) = \frac{1}{k^2 + \mu_0^2 + \Delta m_T^2}\,. \tag{8}$$

The leading contribution to the imaginary part of the self-energy comes from the two loop sunset diagram. It can be expressed as [4]

$$\mathrm{Im}\Sigma_2(\omega_{\mathbf{p}}, \mathbf{p}) = \mathrm{Im}\Sigma^{3BD}(\omega_{\mathbf{p}}, \mathbf{p}) + \mathrm{Im}\Sigma^{LD}(\omega_{\mathbf{p}}, \mathbf{p})\,. \tag{9}$$

$\mathrm{Im}\Sigma^{3BD}$ and $\mathrm{Im}\Sigma^{LD}$ are contribution from the 3-body decay and Landau damping, respectively:

$$\begin{aligned}\mathrm{Im}\Sigma^{3BD}(\omega_{\mathbf{p}}, \mathbf{p}) &= \pi(e^{\frac{\omega_{\mathbf{p}}}{T}} - 1)\int d[\mathbf{k},\mathbf{q}] f_{\mathbf{k}} f_{\mathbf{q}} f_{\mathbf{r}} \\ &\quad \times\delta(\omega_{\mathbf{p}} - E_{\mathbf{k}} - E_{\mathbf{q}} - E_{\mathbf{r}})\,; \qquad (10)\\ \mathrm{Im}\Sigma^{LD}(\omega_{\mathbf{p}}, \mathbf{p}) &= 3\pi(e^{\frac{\omega_{\mathbf{p}}}{T}} - 1)\int d[\mathbf{k},\mathbf{q}](1 + f_{\mathbf{k}}) f_{\mathbf{q}} f_{\mathbf{r}} \\ &\quad \times\delta(\omega_{\mathbf{p}} + E_{\mathbf{k}} - E_{\mathbf{q}} - E_{\mathbf{r}})\,, \qquad (11)\end{aligned}$$

with the same notation as in Ref. [4],

$$\begin{aligned} d[\mathbf{k},\mathbf{q}] &= \frac{\lambda^2\mu^{4\epsilon}}{6}\frac{d^{D-1}\mathbf{k}}{(2\pi)^{D-1}}\frac{d^{D-1}\mathbf{q}}{(2\pi)^{D-1}}\frac{1}{8E_{\mathbf{k}}E_{\mathbf{q}}E_{\mathbf{r}}} && (12)\\ \mathbf{r} &= \mathbf{k} + \mathbf{q} - \mathbf{p}\,, && (13)\\ E_{\mathbf{l}}^2 &= \mathbf{l}^2 + m_p^2\,, \qquad \mathbf{l} = \mathbf{k},\mathbf{q},\mathbf{r}\,, && (14)\\ f_{\mathbf{l}} &= \frac{1}{\exp{(E_{\mathbf{l}}/T)} - 1}\,, \qquad \mathbf{l} = \mathbf{k},\mathbf{q},\mathbf{r}\,. && (15)\end{aligned}$$

The plasmon damping rate with finite momentum can be obtained as [4]

$$\gamma(\sqrt{|\mathbf{p}|^2 + m_p^2}, \mathbf{p}) = \frac{\lambda_0^2 T}{256\pi^3}\frac{1}{z\epsilon(z)} f(z)\,, \tag{16}$$

where

$$\begin{aligned} f(z) &= \int_0^z dx\left[L_2(\xi) + L_2\left(\frac{\xi - \zeta}{\xi(1-\zeta)}\right)\right. \\ &\quad \left. - L_2\left(\frac{\xi - \zeta}{1-\zeta}\right) - L_2\left(\frac{(\xi-\zeta)(1-\xi\zeta)}{\xi(1-\zeta)^2}\right)\right], && (17)\\ z &= \frac{|\mathbf{p}|}{T}, \quad a = \frac{m_p}{T}, \quad \epsilon(z) = \sqrt{z^2 + a^2}, && (18)\\ \xi &= e^{-\epsilon(z)}, \quad \zeta = e^{-\epsilon(x)}\,, && (19)\\ L_2(y) &= -\int_0^y dt\frac{\ln(1-t)}{t}\,. && (20)\end{aligned}$$

For the plasmons at rest ($\mathbf{p} = 0$) the damping rate reduces to

$$\gamma(m_p^2, \mathbf{p} = 0) = \frac{\lambda_0^2 T^2}{1536\pi m_p^2}\,. \tag{21}$$

As $T \to T_c$ the vanishing plasmon mass results in divergent damping rate for the plasmons at rest. This means that the relaxation time becomes shorter and shorter as the critical temperature is approached. This behavior contradicts the critical slowing down law exhibited in the condensed matter systems. We should notice that, in getting above result, the coupling constant λ_0 is considered as a temperature-independent. Actually, in thermal field theory the coupling constant runs with temperature from thermal TRG equation.

As shown in Ref. [7], in the framework of Wilson renormalization group, the TRG equation in real time thermal field theories is deduced as

$$\Lambda\frac{\partial\Gamma_\Lambda[\varphi]}{\partial\Lambda} = \frac{i}{2}\mathrm{Tr}\left[\Lambda\frac{\partial D_\Lambda^{-1}}{\partial\Lambda}\cdot\left(D_\Lambda^{-1} + \frac{\delta^2\Gamma_\Lambda[\varphi]}{\delta\varphi\delta\varphi}\right)^{-1}\right], \tag{22}$$

where Λ is a cut-off introduced in the thermal sector of real-time propagator in Closed Time Path (CPT) formalism [8] by modifying the Bose-Einstein distribution function $N(k_0) = 1/[\exp(k_0/T) - 1]$ as

$$N_\Lambda(k_0) = N(k_0)\theta(|\mathbf{k}| - \Lambda)\,; \tag{23}$$

D_Λ is the tree level propagator with $N_\Lambda(k_0)$ in the CPT formalism; Γ_Λ is the generating function of 1PI vertex function in which the modes with $k > \Lambda$ have been integrated out.

From Eq.(22) the evolution equations for 2-point and 4-point functions can be expressed as

$$\begin{aligned}\Lambda\frac{\partial}{\partial\Lambda}\Gamma_\Lambda^{(2)} &= \frac{1}{2}\mathrm{Tr}[K_\Lambda\Gamma_\Lambda^{(4)}]\,, && (24)\\ \Lambda\frac{\partial}{\partial\Lambda}\Gamma_\Lambda^{(4)} &= -3\mathrm{Tr}[G_\Lambda\Gamma_\Lambda^{(4)}K_\Lambda\Gamma_\Lambda^{(4)}]\,, && (25)\end{aligned}$$

where K_Λ and n-point 1PI vertex function are defined as

$$\begin{aligned} K_\Lambda(k,\varphi) &\equiv -iG_\Lambda\cdot\Lambda\frac{\partial}{\partial\Lambda}D_\Lambda^{-1}\cdot G_\Lambda\,, && (26)\\ \Gamma_\Lambda^{(n)}(k,\varphi) &= \frac{\delta^n\Gamma_\Lambda[\phi]}{\delta\phi_{i_1}\delta\phi_{i_2}\cdots\delta\phi_{i_n}}|_{\phi_{i_1}=\cdots=\phi_{i_n}=\varphi}\,. && (27)\end{aligned}$$

Here G_Λ is the full propagator

$$G_\Lambda^{-1}(k;\varphi) = D_\Lambda^{-1}(k;\varphi) + \Sigma_\Lambda(k;\varphi)\,, \tag{28}$$

self-energy Σ_Λ is 1PI 2-point vertex,

$$\Sigma_\Lambda(k;\varphi) = \Gamma_\Lambda^{(2)}(k;\varphi) \equiv \frac{\delta^2\Gamma_\Lambda[\phi]}{\delta\phi\delta\phi}|_{\phi_1=\phi_2=\varphi}\,. \tag{29}$$

Define 4-point function and cut-off mass as

$$\begin{aligned}\Gamma_\Lambda^{(4)} &= -\lambda_\Lambda - i\eta\,, && (30)\\ m_\Lambda^2 &= \mu_0^2 - \mathrm{Re}\Gamma_\Lambda^{(2)}\,, && (31)\end{aligned}$$

and substitute them into Eqs.(24) and (25), the TRG equations for the plasmon mass and the coupling constant are deduced as

$$\Lambda\frac{\partial}{\partial\Lambda}m_\Lambda^2 = -\frac{\Lambda^3}{4\pi^2}\frac{N(\omega_\Lambda)}{\omega_\Lambda}\lambda_\Lambda\,, \tag{32}$$

$$\Lambda\frac{\partial}{\partial\Lambda}\lambda_\Lambda = -3\frac{\Lambda^3}{4\pi^2}\left(\frac{d}{dm_\Lambda^2}\frac{N(\omega_\Lambda)}{\omega_\Lambda}\right)\lambda_\Lambda^2\,, \tag{33}$$

where $\omega_\Lambda = \sqrt{\Lambda^2 + m_\Lambda^2}$.

The initial conditions for the evolution equations (32) and (33)are given at a scale $\Lambda = \Lambda_0 \gg T$,

$$m_{\Lambda_0}^2 = \mu_0^2\,, \qquad \lambda_{\Lambda_0} = \lambda_0\,. \tag{34}$$

Due to the exponential suppression in the Bose-Einstein distribution function, it is enough to take $\Lambda_0 = 10T$.

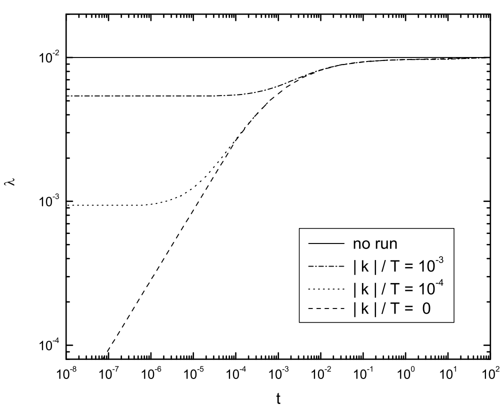

FIG. 1. The running of the coupling constant versus dimensional variable $t = (T - T_c)/T_c$ given by the TRG equations. The dashed, dotted and dash-dotted curves are for $|\mathbf{k}|/T$=0, 10^{-4} and 10^{-3}, respectively. The solid line is for no-running coupling constant fixed to 0.01.

For the moving plasmon with finite momentum $|\mathbf{k}|$, the non-vanishing external momentum $|\mathbf{k}|$ provides an IR cutoff to the TRG running, so the running will effectively stop as Λ is of the order of $|\mathbf{k}|$. When $\Lambda \to |\mathbf{k}|$, we take the solutions $m^2_{\Lambda=|\mathbf{k}|}$ and $\lambda_{\Lambda=|\mathbf{k}|}$ as the results we are expecting for. For the plasmon with zero and finite external momenta, the coupling constants $\lambda_{\Lambda=|\mathbf{k}|}$ versus dimensionless variable $t = (T - T_c)/T_c$ are illustrated in Fig.1. The numerical results show indeed that, because of the IR cutoff provided by the external finite momentum, the momentum-dependent coupling constant stops running with temperature and keeps a constant in the critical region. As shown in Fig.1, the region keeping the momentum-dependent coupling constant as a no-running constant increases with increasing the external momentum. In the following, we will show that the above features of the running coupling constant will change the critical behavior for the plasmon with finite external momentum at the critical point by comparing to that for the plasmon with vanishing external momentum.

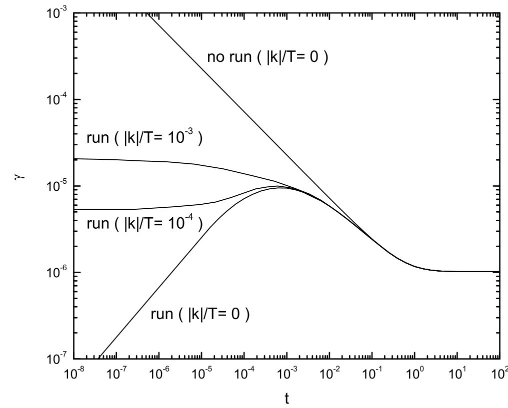

FIG. 2. The plasmon damping rate versus dimensional variable $t = (T - T_c)/T_c$ for different momenta. The upper and lower curves are for plasmon at rest with no-running and running coupling constant, respectively. The middle two curves result from the momemtum-dependent running coupling constant for the plasmon with momentum $|\mathbf{k}|/T = 10^{-3}, 10^{-4}$, respectively.

Substituting running coupling constant into Eq.(16), we get the damping rate versus temperature as illustrated in Fig.2. For the damping rate of the plasmon at rest, the result in Ref. [5] is reproduced by the lower curve in Fig.2. It is clear that the damping rate goes to zero instead of infinity (divergence) as temperature approaches to critical point, by taking into account the running coupling constant with temperature from TRG equation. This corresponds to that the relaxation time goes to infinity at critical point, the behavior is consistent with the critical slowing down law. For the plasmons with finite momentum the damping rates versus temperature are illustrated by the middle two curves in Fig. 2 for the external momentum $|\mathbf{k}|/T = 10^{-3}$ and 10^{-4}, respectively. For much small external momentum $|\mathbf{k}|/T = 10^{-4}$, the momentum-dependent coupling constant can run into the critical region, the damping rate goes up at first as temperature approaches gradually the critical region, and then goes down with coupling constant running down as the temperature enters the critical region. After entering the critical region, tendency of the plasmon damping rate is opposite to that obtained from no-running coupling constant, and the relaxation time gets longer and longer, which is then consistent with the critical slowing down law. As the critical point is approached further, the momentum-dependent coupling constant stops running with temperature, the plasmon damping rate stops decreasing and doesn't vanish, the critical slowing down phenomenon doesn't occur at the critical point. As $|\mathbf{k}|/T = 10^{-3}$, the momentum-dependent coupling constant can't run into the critical region. Although the damping rate $\gamma_{\mathbf{k}}(T)$ is no longer divergent as the critical temperature is approached, which is different from the critical behavior of the plasmon with zero external

momentum for no-running coupling constant, but the damping rate increases still with approaching the critical temperature. This means that the relaxation time gets shorter and shorter as temperature approaches critical point. the critical slowing down phenomenon doesn't take place completely. We notice that the critical slowing down phenomenon is an inherently IR effect, which takes places only in the double limit for external momentum $|\mathbf{k}| \to 0$ and the cut-off $\Lambda \to 0$.

In summary, based on the TRG equations in the CTP formalism, we investigate the critical behavior of the damping rate of the plasmon with finite momentum in ϕ^4 theory with spontaneous breaking symmetry. For the plasmon with vanishing external momentum, the no-running coupling constant leads to that the critical behavior is against the critical slowing down law, this violation can be cured by taking into account the running of the coupling constant with temperature from the TRG equations. For the plasmon with finite external momentum, the IR cutoff provided by the external finite momentum leads to that the momentum-dependent coupling constant stops running with temperature in the vicinity of the critical point. Only for enough smaller external momentum, the momentum-dependent coupling constant can run into the critical region, we can see the tendency of critical slowing down phenomenon. Nearby the critical point, the momentum-dependent coupling constant stops running and keeps a constant, the critical slowing down phenomenon doesn't take place for the plasmon with finite external momentum because the critical slowing down phenomenon is an inherently IR effect. Our recent papers show that the shear viscosity of the thermal scalar field is closely related to the damping rate of the plasmons with finite momentum [9]. We anticipate that, by taking into account the TGR equation, the damping rate discussed in this paper will result in important effects on the transport properties of the thermal scalar field. The further work has been in progress.

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